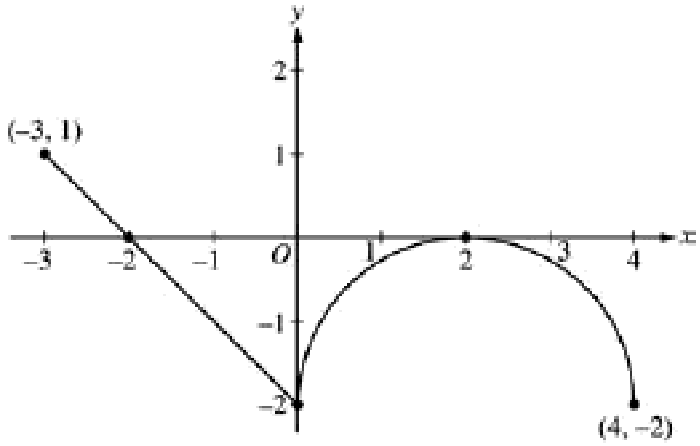
<u>AP Calculus AB Problem Wednesday, December 13, 2006</u>



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with f(0) = 3. The graph of f ', the derivative of f, consists of one line segment and a semicircle, as shown above.

a. On what intervals, if any, is f increasing? Justify your answer.

The function f is increasing on [-3,-2] since f ' > 0 for $-3 \le x < -2$.

b. Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

The x-coordinates of the points of inflection are x = 0 and x = 2. f ' changes from decreasing to increasing at x = 0 and from increasing to decreasing at x = 2.

c. Find an equation for the line tangent to the graph of f at the point (0,3).

The tangent line contains the point (0,3) and has slope -2 since f '(0) = -2. The equation is y = -2x + 3.

d. Find f(-3) and f(4). Show the work that leads to your answers.

We know that f(0) = 3

$$f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt = \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}.$$

$$f(-3) = f(0) + \frac{3}{2} = \boxed{\frac{9}{2}}.$$

$$f(4) - f(0) = \int_{0}^{4} f'(t)\,dt = -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$$

$$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$$